Polynomials: zeros and y-intercept

Graphing Polynomial Functions

Things you need to know:

1) Degree

2) Leading Coefficient

3) End Behavior

4) extrema (max + min)

5) y - intercept

6) zeros and multiplicity

Polynomials: zeros and y-intercept

Finding the y - intercept

To find the y - intercept, plug in 0 for x. Notation: (0 , y)

1) $f(x) = -x^3 + 4x^2 - 3x + 3$

(0, 3)

2) $y = 6x^2 - 3x + 5$

(0, 5)

Practice:

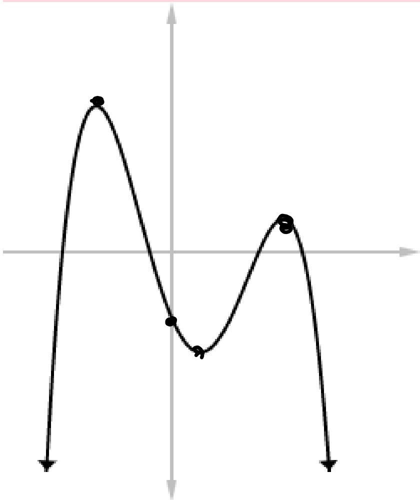
1) $g(x) = 4x^6 + 5x^4 - 3x^2$

(0, 0)

2) $y = 2x + 15$

(0, 15)

Polynomials: zeros and y-intercept



Which of the following statements are true about f(x) graphed to the left?

a) f(x) is a quadratic F

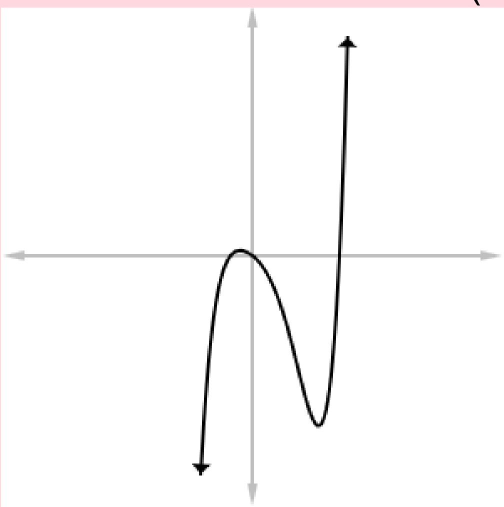
b) the degree of f(x) is even T

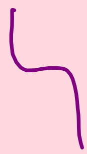
c) the constant term of f(x) is positive F

d) f(x) has a positive leading coefficient F

Polynomials: zeros and y-intercept

Which of the following statements are true about f(x) graphed to the left?





a) f(x) is a quartic F

b) the degree of f(x) is even F

c) the constant term of f(x) is zero T

d) f(x) has a positive leading coefficient T

$-3x^3-4x^2$

Polynomials: zeros and y-intercept

Roots, zeros, solutions, x-intercepts

Roots are the numbers that make expression zero.

Expressions have **ROOTS.**

$$(x-3)(x-5)$$

Polynomials: zeros and y-intercept

Roots, zeros, solutions, x-intercepts

<u>**Zeros**</u> are the numbers that make function equal to zero.

Functions have

ZEROS.

$$f(x)=(x-3)(x-5)$$

Polynomials: zeros and y-intercept

Roots, zeros, solutions, x-intercepts

Solution are the answers to an equation.

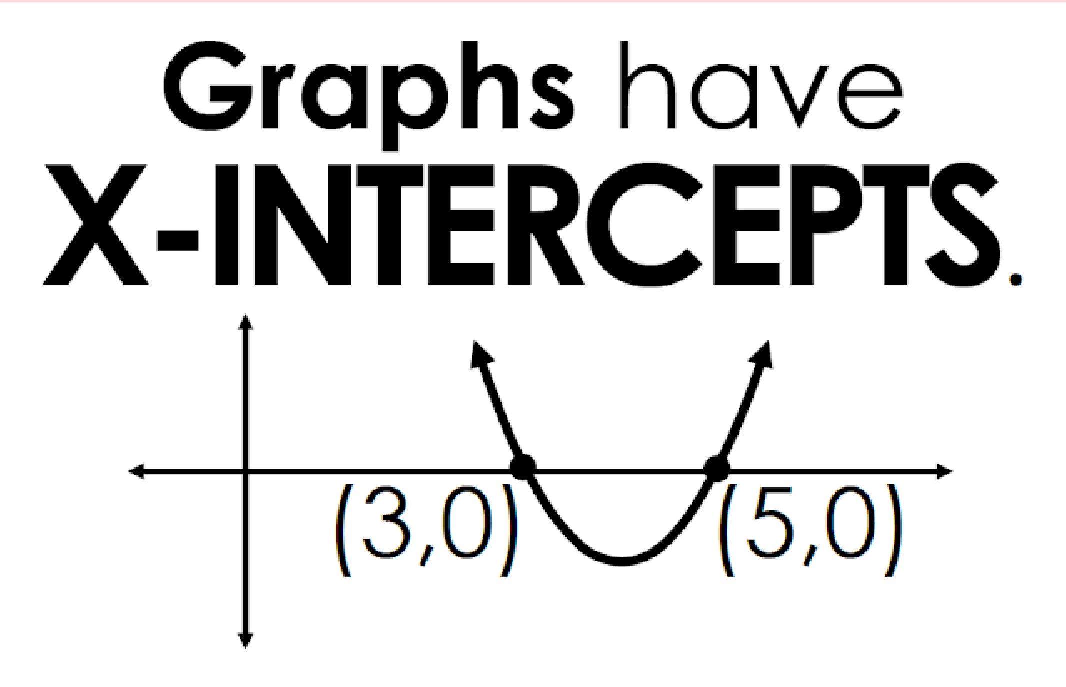
Equations have **SOLUTIONS.**

$$(x-3)(x-5)=0$$

Polynomials: zeros and y-intercept

Roots, zeros, solutions, x-intercepts

x-intercepts are the points where the graph crosses the x-axis.



Polynomials: zeros and y-intercept

Roots, zeros, solutions, x-intercepts

Big takeaway

x - intercepts are

the **real zeros** of a function

the **real solutions** of an equation

the **real roots** of an expression

Polynomials: zeros and y-intercept

The Fundamental theorem of Algebra

Fundamental theorem of algebra
(simplified version)

- Every polynomial of degree n will have n roots counting both real and non real roots up to their multiplicity.

  (We say a polynomial has degree n where n is the largest exponent of x.

- Ex. $f(x) = x^5 + x^4 + x^3 + x^2 + x + 1$ has 5 total zeros.

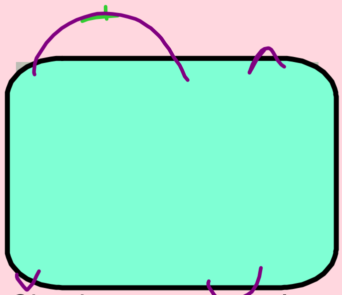
  $f(x) = ax^3 + bx^2 + cx + d$ has 3 total zeros

Polynomials: zeros and y-intercept

Find all the zeros of the following function.

$4x^5$ Deg. 5

$$f(x) = (x^2 + 16)(x - 5)(x + 9)(4x - 7)$$



Check on your calculator

$x^2 + 16 = 0$ $x - 5 = 0$ $x + 9 = 0$ $4x - 7 = 0$

$x^2 = -16$ $x = 5$ $x = -9$ $x = \frac{7}{4}$

$x = \pm 4i$

x-int.

Polynomials: zeros and y-intercept

Find all the solutions of the following equation.

D: 5

$$x(x - 3)(2x + 3)(x + 6)(x + 1) = 0$$

$x = 0$ $x = 3$ $x = -1$

$x = -6$ $x = \frac{-3}{2}$

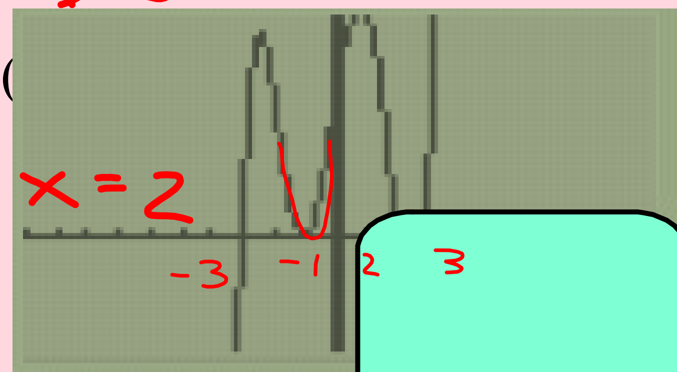
Check on your calculator

Polynomials: zeros and y-intercept

Find all the x - intercepts of the graph of the following function. D: 5

(x+1)(x+1)

$f(x) = (x + 1)^2(x^2 - 9)($



x = -1 x = ± 3 x = 2

mult. 2

Polynomials: zeros and y-intercept

Multiplicity of Zeros

Multiplicity is the number of times that a **zero** appears.

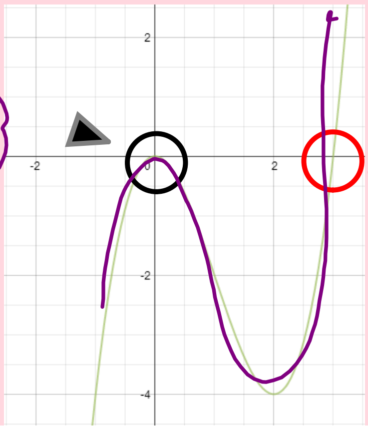
D: 3

$$f(x) = x^2(x - 3)$$

$x^3 - 3x^2$

Zeros: [redacted]

Multiplicity: [redacted]



Even multiplicity
"Bounces"

touching Point

Odd multiplicity
"Through" *x*-axis

Polynomials: zeros and y-intercept

Write $f(x) = x^3 - 6x^2 + 9x$ in factored form.

$f(x) = x(x^2 - 6x + 9)$
$= x(x-3)(x-3)$
$= x(x-3)^2$

y-int: (0,0)
D: 3
LC: +1
↓

What are the zeros and their multiplicity?

$x = 0$
mult. 1
go through

$x = 3$
mult. 2
bounce

Polynomials: zeros and y-intercept

Sketch the graph of $f(x) = x^3 - 6x^2 + 9x$

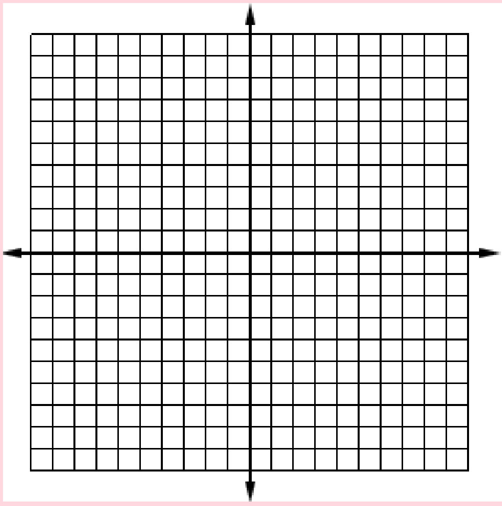
Degree:

Leading coefficient:

End Behavior:

Turning Points:

Zeros and multiplicity:



Polynomials: zeros and y-intercept

Multiplicity of zeros. Find all the zeros and state the multiplicity. Tell whether they bounce or pass through.

$$f(x) = 3(x + 5)^2(x^2 - 4)(3x - 5)^3$$

D: 7
LC: + ↑ ↓

$x = -5$ mult: 2 bounce

$x = \pm 2$ mult: 1 goes through

$x = \frac{5}{3}$ mult: 3 goes through

Polynomials: zeros and y-intercept

Sketch the graph of each of the following.

Examples:

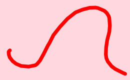
$y = -3(x - 2)^2(3x - 2)$

D: 3 LC: − ↑ ↓

zeros: 2 $\frac{2}{3}$

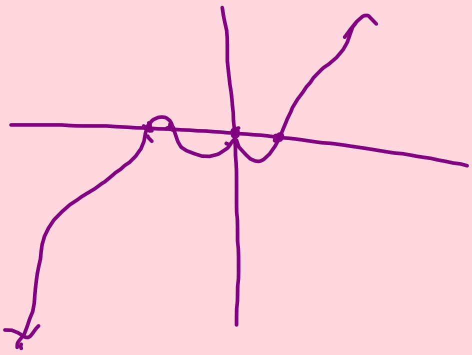
mult: 2 1

bounce go through



Practice:

$y = 5x^4(x + 3)^5 (x^2 - 4)$



$y = 4x^3(2x + 1)^4 (x^2 + 4)$

$y = - x^4 + 2x^2 + 35$

Polynomials: zeros and y-intercept

Sketch the graph of each of the following.

Examples:

$y = x^4 + x^2 - 6$

$y = x^6 + 2x^4 - 16x^2 - 32$

Practice:

$y = - x^4 + 2x^2 + 35$

$y = x^6 - x^4 - 9x^2 + 9$

Polynomials: zeros and y-intercept

A polynomial function has zeros at - 6 (mulitplicity of 2) and 2 (mulitplicity of 1). Write a polynomial in standard form that could represent this function.

$f(x) = (x+6)^2 (x-2)$

$= (x+6)(x+6)(x-2)$

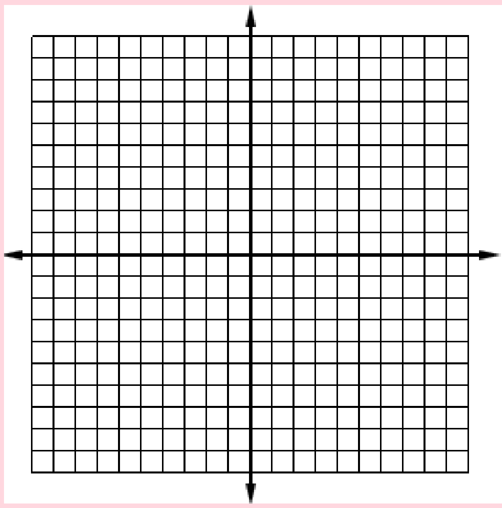
Polynomials: zeros and y-intercept

Finding Zeros in the calculator

$f(x) = x^3 - 10x^2 + 27x - 18$

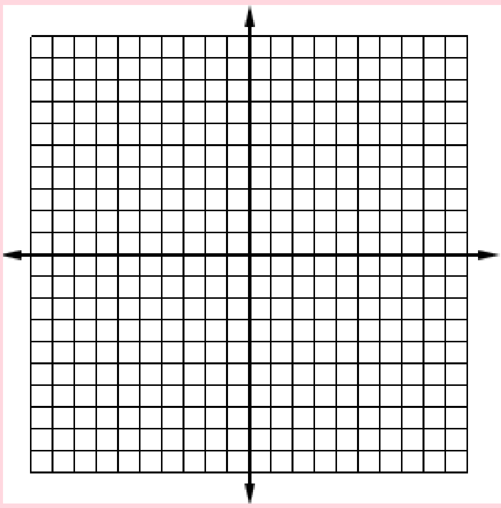
Polynomials: zeros and y-intercept

$f(x) = x^3 + 6x^2 + 8x$



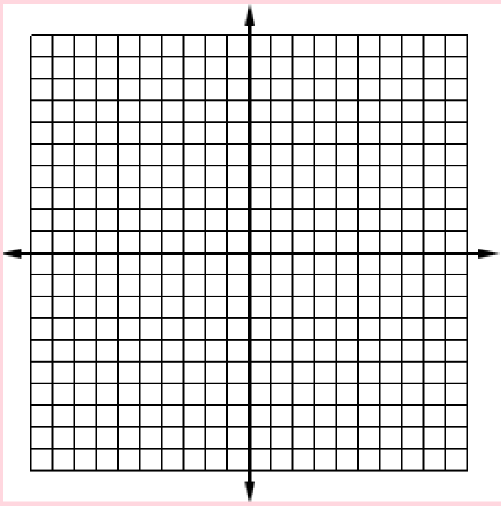
Polynomials: zeros and y-intercept

$f(x) = -x^3 - 10x^2 + 8x + 80$



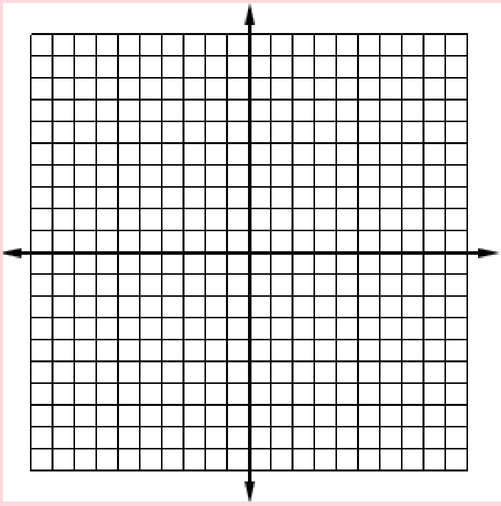
Polynomials: zeros and y-intercept

$f(x) = -x^4 - 9x^3 - 2x^2 - 18x$



Polynomials: zeros and y-intercept

$f(x) = x^4 - 9$



Polynomials: zeros and y-intercept

Polynomials: zeros and y-intercept