Imaginary Roots and Conjugate Root Theorem

Bellwork:

$a^3 - b^3 = (a-b)(a^2+ab+b^2)$

Find all the zeros of

$$f(x) = x^3 - 8$$

D: 3

Zeros: 3

$0 = x^3 - 8$

$8 = x^3$

$x = 2$

x-int.

$0 = (x-2)(x^2+2x+4)$

$x - 2 = 0$

$x = 2$

$x^2+2x+4 = 0$

$x: \dfrac{-2 \pm \sqrt{4-4(1)(4)}}{2}$

$= \dfrac{-2 \pm \sqrt{4-16}}{2}$

$= \dfrac{-2 \pm \sqrt{-12}}{2} = \dfrac{-2 \pm 2i\sqrt{3}}{2} = \boxed{-1 \pm i\sqrt{3}}$

Imaginary Roots and Conjugate Root Theorem

Find all the zeros of

$$f(x) = x^3 + 2x^2 + 4x + 8$$

D: 3
Z: 3

$0 = x^3 + 2x^2 + 4x + 8$

$0 = x^2(x+2) + 4(x+2)$

$0 = (x+2)(x^2+4)$

$x + 2 = 0$ $\quad$ $x^2 + 4 = 0$

$x = -2$ $\quad$ $x^2 = -4$

$x = \pm\sqrt{-4} = \pm 2i$

Imaginary Roots and Conjugate Root Theorem

take note

Theorem **Conjugate Root Theorem**

If $P(x)$ is a polynomial with *rational* coefficients, then irrational roots of $P(x) = 0$ that have the form $a + \sqrt{b}$ occur in conjugate pairs. That is, if $a + \sqrt{b}$ is an irrational root with a and b rational, then $a - \sqrt{b}$ is also a root.

If $P(x)$ is a polynomial with *real* coefficients, then the complex roots of $P(x) = 0$ occur in conjugate pairs. That is, if $a + bi$ is a complex root with a and b real, then $a - bi$ is also a root.

Summary:

if $2+\sqrt{3}$ is a root then $2-\sqrt{3}$ is one also

if $1+6i$ is a root then $1-6i$ is one also

Imaginary Roots and Conjugate Root Theorem

How many and what types of roots. Example:

$10x^6 + 7x^3 + 4x^2 + 17$

Total: 6

imaginary: 0, 2, 4, 6

real: 6, 4, 2, 0

Imaginary Roots and Conjugate Root Theorem

Write a polynomial function of least degree with integral coefficients that has the given zeros.

zeros: 4/3, - 2 mult. 2

$(3x-4)(x+2)^2$

$x+2=0$
$x=-2$

$3x-4=0$
$x=4/3$

$= (3x-4)(x+2)(x+2)$

$= (3x-4)(x^2+4x+4)$

$= 3x^3+12x^2+12x-4x^2-16x-16$

$\boxed{= 3x^3+8x^2-4x-16}$

Imaginary Roots and Conjugate Root Theorem

Write a polynomial function of least degree with integral coefficients that has the given zeros.

zeros: $-2i$, $3 + \sqrt{2}$

Imaginary Roots and Conjugate Root Theorem

Write a polynomial function of least degree with integral coefficients that has the given zeros.

zeros: -2/5, 3i , -3i

$(5x+2)\underbrace{(x-3i)(x+3i)}$

$-9(-1)$

$(5x+2)(x^2+\cancel{3ix}-\cancel{3ix}-9i^2)$

$(5x+2)(x^2+9)$

$=5x^3+45x+2x^2+18$

$=5x^3+2x^2+45x+18$

Imaginary Roots and Conjugate Root Theorem

Write a polynomial function of least degree with integral coefficients that has the given zeros.

zeros: $\underbrace{3 + 2\sqrt{6}}_{3-2\sqrt{6}}$, $\underbrace{-1+ 3i}_{-1-3i}$

$(x-(3+2\sqrt{6}))(x-(3-2\sqrt{6}))(x-(-1+3i))(x-(-1-3i))$

$(x-3-2\sqrt{6})(x-3+2\sqrt{6})(x+1-3i)(x+1+3i)$

$= x^2 - 3x + \cancel{2x\sqrt{6}} - 3x + 9 + \cancel{6\sqrt{6}} - \cancel{2x\sqrt{6}} + \cancel{6\sqrt{6}} - 4\sqrt{36}$

-24

$= (x^2 - 6x - 15)$

$= x^2 + x + \cancel{3ix} + x + 1 + \cancel{3i} - \cancel{3ix} - \cancel{3i} - 9i^2 \leftarrow +9$

$= x^2 + 2x + 10$

$= (x^2 - 6x - 15)(x^2 + 2x + 10)$

Imaginary Roots and Conjugate Root Theorem

Write a polynomial function in standard form of least degree with integral coefficients that has the given zeros.

1) -6, 3, and 1 - 5i

2) $4 + \sqrt{5}$ and 8

3) - 7i and $2 - \sqrt{11}$

$(x+6)(x-3)(x-1+5i)(x-1-5i)$

$(x-4+\sqrt{5})(x-4-\sqrt{5})(x-8)$

$(x+7i)(x-7i)(x-2+\sqrt{11})$
$(x-2-\sqrt{11})$

Answers:

1) $x^4 + x^3 + 2x^2 + 114x - 468$

2) $x^3 - 16x^2 + 75x - 88$

3) $x^4 - 4x^3 + 38x^2 - 196x - 539$

Imaginary Roots and Conjugate Root Theorem

Let $f(x) = x^7 - 4x^3 + 7x^2 + x - 5$

Given that $3 + 2i$ is one solution to $f(x)$ what can you tell me about the other solutions? $3 - 2i$

How many could be real?

How many could be imaginary?

Which solutions do you know for sure?

Imaginary Roots and Conjugate Root Theorem

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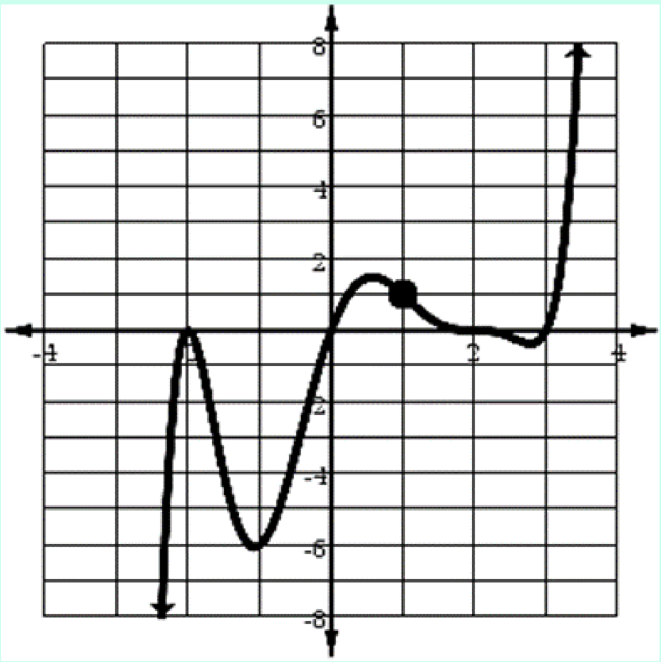
Imaginary Roots and Conjugate Root Theorem

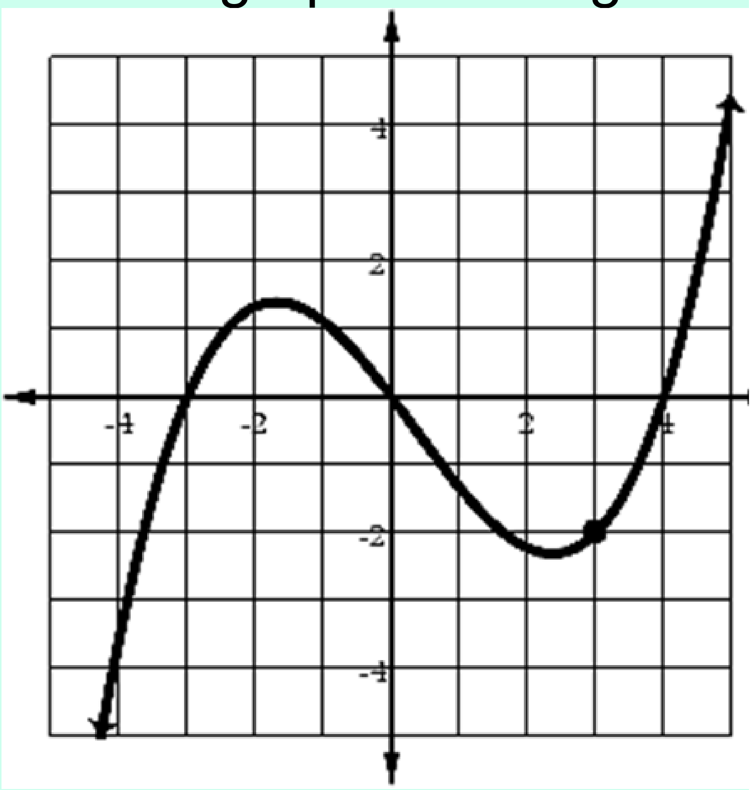
Write the a function for the graph to the right.



Imaginary Roots and Conjugate Root Theorem

Write the a function for the graph to the right.

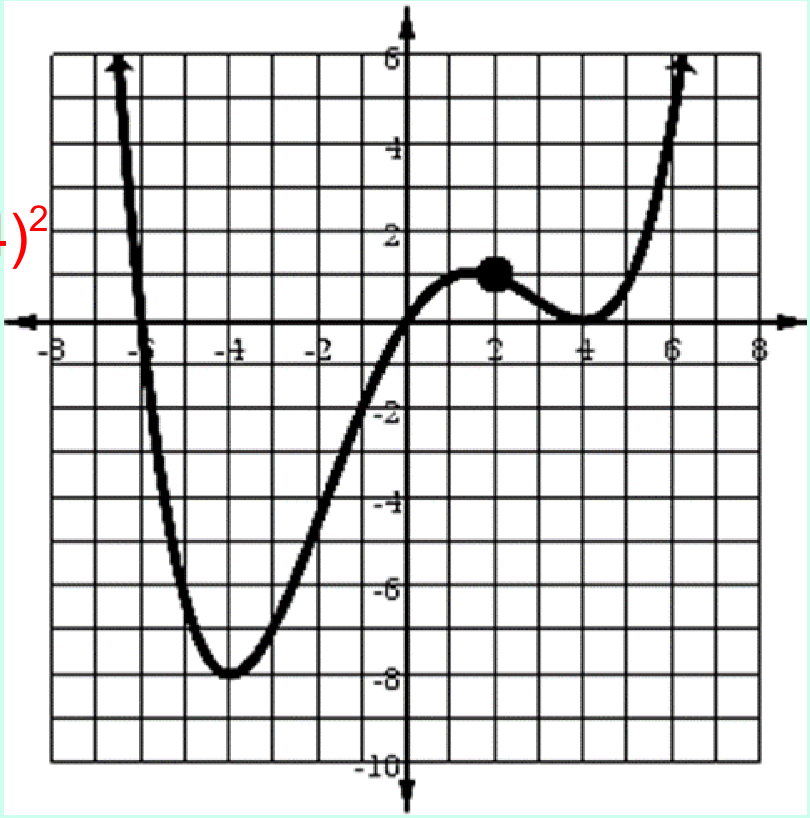
$f(x) = x(x + 3)(x - 4)$



Imaginary Roots and Conjugate Root Theorem

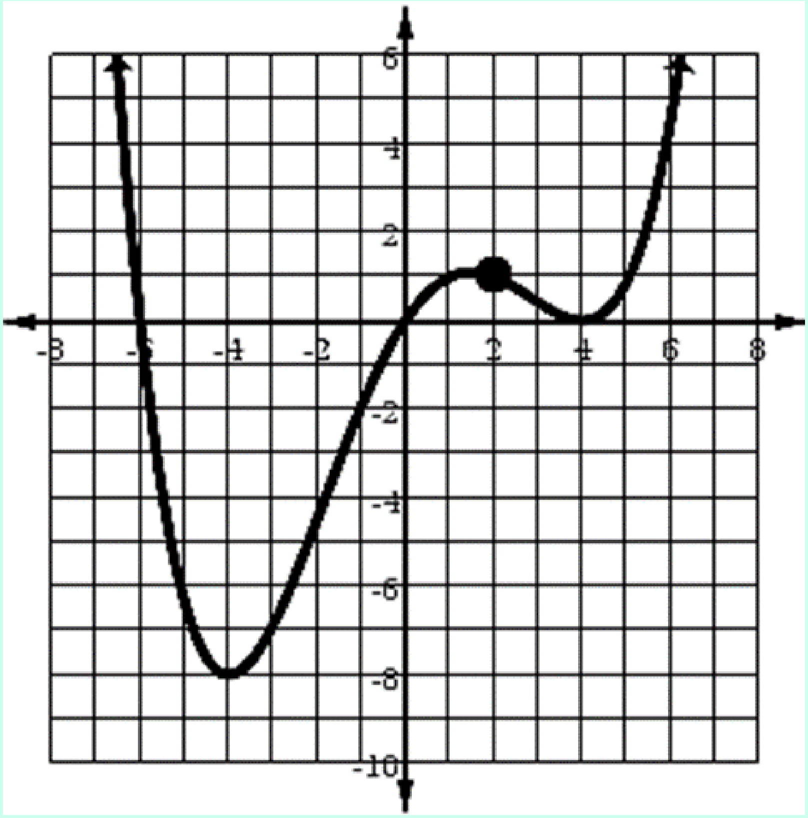
Write the a function for the graph to the right.

$f(x) = x(x + 6)(x - 4)^2$



Imaginary Roots and Conjugate Root Theorem

State the intervals of increasing and decreasing.



Increasing:

(-4, 2)(4,∞)

Decreasing:

(-∞,-4)(2, 4)