Solving Polynomials Review

Bellwork:

1) Factor completely $8x^4 + 10x^2 - 3$

$(4x^2 - 1)(2x^2 + 3)$

$(2x - 1)(2x + 1)(2x^2 + 3)$

2) Factor completely $4x^3 - x^2 - 4x + 1$

$x^2(4x - 1) - 1(4x - 1)$

$(x^2 - 1)(4x - 1)$

$(x - 1)(x + 1)(4x - 1)$

Solving Polynomials Review

Rational Root Theorem: : For every polynomial of the form

$P(x) = ax^n + bx^{n-1} + ... + c$, where a,b and c are integers and $a,c \neq 0$ if P(x) has rational a root of the reduced form p/q, then p is a factor of the constant term c and q is a factor of the leading coefficient a.

When asked for a list of all possible rational roots

Solving Polynomials Review

List all possible rational roots of

$y = 5x^4 + 8x^2 - 4$

$\frac{1}{5}$ $\frac{2}{4}$

± 2 $\pm \frac{2}{5}$

± 1 $\pm \frac{1}{5}$

± 4 $\pm \frac{4}{5}$

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List all possible rational roots of

$y = 12x^3 + 4x^2 - 6x - 4$

$\frac{2}{3}$ $\frac{1}{4}$ $\frac{1}{6}$ $\frac{1}{2}$ $\frac{1}{12}$

$\pm\frac{1}{3}$ $\pm\frac{1}{4}$ $\pm\frac{1}{10}$ $\pm\frac{1}{2}$ ± 1 $\pm\frac{1}{12}$ $\pm\frac{2}{3}$ ± 2

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List all possible rational roots of

$y = 10x^4 - 40$

1, 10, 2, 5

4, 10, 8, 5, 1, 40, 2, 20

$\pm 4 \quad \pm\frac{2}{5} \quad \pm 2 \quad \pm\frac{4}{5}$

$\pm 10 \quad \pm 1 \quad \pm 5 \quad \pm 8$

$\pm\frac{8}{5} \quad \pm\frac{1}{2} \quad \pm\frac{5}{2} \quad \pm\frac{1}{10}$

$\pm\frac{1}{5} \quad \pm 40 \quad \pm 20$

Solving Polynomials Review

List all possible rational roots of

$y = 5x^4 + 8x^2 - 4$

$0 = 5x^4 + 8x^2 - 4$

$0 = (5x^2 - 2)(x^2 + 2)$

$5x^2 - 2 = 0 \quad x^2 + 2 = 0$

$x^2 = \frac{2}{5} \quad x^2 = -2$

$x = \pm\sqrt{\frac{2}{5}} \quad x = \pm\sqrt{-2}$

zeros: 4

real: 2, 4, 0

imag: 2, 0, 4

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First discuss the nature of the solution and then find all the solutions of the equation below.

$y = 12x^3 + 4x^2 - 6x - 4$

$0 = 2(6x^3 + 2x^2 - 3x - 2)$

$0 = 2(2x^2(3x+1) - 1(3x+1))$

$0 = 2(3x+1)(2x^2-1)$

$2 \neq 0$ $\quad 3x+1=0$ $\quad 2x^2-1=0$

$x = -\frac{1}{3}$ $\quad x^2 = \frac{1}{2}$

$x = \pm\sqrt{\frac{1}{2}}$

zeros: 3

real: 3, 1

img: 0, 2

Solving Polynomials Review

First discuss the nature of the solution and then find all the solutions of the equation below.

$y = 10x^4 - 40$

$0 = 10(x^4 - 4)$

$0 = 10(x^2+2)(x^2-2)$

~~$10 = 0$~~ $x^2+2=0$ $x^2-2=0$

$x^2 = -2$ $x^2 = 2$

$x = \pm i\sqrt{2}$ $x = \pm\sqrt{2}$

zeros: 4

real: 2, 4, 0

imag: 2, 0, 4

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Solve.

$-64x^3 + 125 = 0$

$(-4x + 5)(16x^2 + 20x + 25) = 0$

$-4x + 5 = 0$

$x = \frac{5}{4}$

$16x^2 + 20x + 25 = 0$

$x = \frac{-20 \pm \sqrt{400 - 4(16)(25)}}{32}$

$x = \frac{-20 \pm \sqrt{-1200}}{32}$

$x = \frac{-20 \pm 20i\sqrt{3}}{32} = \frac{-5 \pm 5i\sqrt{3}}{8}$

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Solve

$2x^3 - 13x^2 + 20x = 0$

$x(2x^2 - 13x + 20) = 0$

$x(2x - 5)(x - 4)$

$x = 0 \quad x = \frac{5}{2} \quad x = 4$

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Solve

$x^3 + x = 0$

Solving Polynomials Review

Solve

$4x^3 - 7x^2 = 3x^2$

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Solve

$x^4 - 64 = 0$

Solving Polynomials Review

Solve

$5x^5 - 80x = 0$

Solving Polynomials Review

Solve

$8x^5 + 5x^2 = 4x^2$

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Solve

$16x^3 = 54$

Solving Polynomials Review

Solve

$x^4 + x^2 - 42 = 0$

Test Questions

1) Which of the equations below has roots at 2i and - 3 + i?

A) $x^2 + ix + 20$

B) $3x^2 - 5x + 10$

C) $x^4 + 6x^3 + 14x^2 + 24x + 40$

D) $x^3 - 3x^2 + 7x + 40$

2) Given that a cubic polynomial has a root of 3 + 2i, what can you tell about the remaining roots?

A) 1 real and 2 imaginary

B) 2 real

C) 2 imaginary

D) 1 real and 1 imaginary

3) Solve the following by any method:

$3x^2 - 4x - 7 = 0$

Solving Polynomials Review

Write a polynomial equation that has roots at 0 multipliciy of 2, and $2 + \sqrt{3}$